DETERMINATION OF THE ELEVATOR POWER AND THE DAMPING IN PITCH OF THE CESSNA 140 AIRPLANE FROM FLIGHT TESTS

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SUMMARY

Presented herein are the results of a flight test program conducted to determine two of the longitudinal stability derivatives of the Cessna 140 airplane. These derivatives are the elevator power, C_{m_δ} , and the damping in pitch, $C_{m_{d\theta}}$. The flight test techniques employed to obtain these derivatives are outlined. The values of C_{m_δ} and $C_{m_{d\theta}}$ thus obtained are compared with the values of these parameters computed from the theoretical formulae as a means of evaluating the steady state flight test procedures employed. Sources of error which arise in the experimental determination of the derivatives are analyzed and techniques of minimizing these errors are indicated.

OBJECTIVE

The objective of this investigation has been the determination of two longitudinal stability derivatives -- the elevator power (C_{m_δ}) and the damping in pitch ($C_{m_{d\theta}}$) -- of a typical light airplane, the Cessna 140. By comparing the experimental values of these derivatives derived from flight test data with the values calculated by means of the theoretical approach, it is hoped that some evaluation of the flight test techniques employed to obtain the derivatives can be made. The results may be of some value to the light airplane designer in determining the required elevator power for airplanes with high damping in pitch.

DATE AND PLACE OF THE INVESTIGATION

The investigation was conducted under the direction of the Princeton University Aeronautical Engineering Department at Princeton Airport, Princeton, New Jersey, from January to May, 1950.

INTRODUCTION

In recent years the increasing speeds and performance of aircraft have necessarily focused much attention to the stability and control problems involved in securing desirable handling qualities of the airplane in high speed flight. In the past, much of the necessary stability and control data could be successfully predicted with good accuracy from wind tunnel tests. Today, however, the high subsonic, transonic, and supersonic speeds under consideration definitely limit the usefulness of the wind tunnel in this field. The difficulties of design and construction and the prohibitive costs of such tunnels, coupled with the inaccuracies of small scale models, force the aerodynamicist to turn to other means to obtain stability and control data. The other possibility is the airplane itself and this has, in fact, been the modern approach to the problem.

In general, it is necessary to deduce the airplane's aerodynamic characteristics from the response to its controls. Much study is now being given to the so-called frequency response of the airplane as a means of determining the stability parameters. In this approach the airplane is oscillated by an oscillatory input to a particular control and its steady state frequency response is recorded. In some cases the frequency response may be deduced from the transient response to a pulsed input to the control. The aerodynamic parameters are then calculated from the frequency response data. While these methods will undoubtedly be of increasing usefulness in the future, they are at present somewhat limited by the need for precise instrumentation and accurate data reduction techniques. Exact knowledge of the airplane's mass and inertia characteristics is required and this in itself is no small problem.

Where the flight duration of the airplane is not limited severely, as in the case of practically all present day subsonic aircraft, certain flight testing techniques for obtaining some of the stability parameters have been developed which are much simpler in theory and execution than the frequency response methods. In many cases these techniques are not only more accurate but their value is enhanced by lower cost, simpler instrumentation, and more precise data reduction. It is with some of these procedures that this paper will deal.

The techniques under consideration involve the deduction of the airplane's elevator power and its damping in pitch from the so-called trim curves taken in unaccelerated and accelerated flight. As a means of evaluating the methods, flight tests of this type were conducted on a Cessna 140 airplane, and these two stability derivatives were computed from the results. Comparison with results obtained in computing the derivatives from the theoretical equations should yield some insight into the accuracy that may be expected in determining these derivatives from flight tests in this manner.

DESCRIPTION OF THE AIRPLANE

The Cessna 140 is a single engine, high wing, two place, personal type monoplane with external bracing and fixed, conventional landing gear. The wing is rectangular with rounded tips. It has the normal configuration, single vertical tail, Frieze type ailerons, and is equipped with trailing edge plain flaps.

The airplane is of semi-monocoque all metal construction except the wings which are fabric covered. All control surfaces are of metal. Figure IV-1 of the appendix illustrates the appearance of the airplane as instrumented for these flight tests.

The following general specifications and dimensions are taken from the drawings and reports of the manufacturer except in the case where they have been modified by the installation of special equipment. In these cases both the old and the new dimension or specification is given.

Airplane, general

Manufacturer	Cessna Aircraft, Co.
Type	140
Recommended gross weight	1450 lbs.
Center of gravity range	
forward limit	22.8% mac.
aft limit	30.0% mac.
Overall length	256.50 in.
Height	74.25 in.
Maximum allowable maneuvering load factor	
gross weight 1460 lbs.	+4.57 to -2.26
flaps down 40°	+1.97 to -2.26

Wing

Airfoil section	NACA 2412
Span	394 in.
Area (total)	159.29 sq. ft.
Area (less ailerons)	145.21 sq. ft.
Aspect ratio	6.76
Taper ratio	1.0
Chord	60.5 in.
Mean aerodynamic chord	
Length	59.02 in.
Distance of leading edge back of nose reference datum line	56.58 in.
Incidence	+1°
Dihedral	+1°

Aileron	
Type	Modified Frieze
Area	14.08 sq. ft.
Span	74 in.
Chord	14 in.
Travel	
up (from neutral)	22°
down (from neutral)	14°
Wing flaps	
Type	Plain, Trailing edge
Area	8.736 sq. ft.
Span	78.625 in.
Chord	8.0 in.
Travel (down)	40°
Horizontal tail surface	
Airfoil section	NACA 0009
Area (including elevators)	24.35 sq. ft.
Span	106 in.
Maximum chord	41.4 in.
Incidence	-2.5°
Dihedral	0
Elevator area (total, including tab)	9.66 sq. ft.
Elevator span	106 in.
Elevator travel	
up (from streamline with stabilizer)	20°
down (from streamline with stabilizer)	20°

Elevator trim tab area	0.695 sq. ft.
Elevator trim tab span	36 in.
Elevator trim tab mean chord	5.20 in.
Elevator trim tab travel	
up (from elevator trailing edge)	6°
down (from elevator trailing edge)	33°
<u>Vertical tail surface</u>	
Area	12.420 sq. ft.
Fin area	6.668 sq. ft.
Span (to fuselage center line)	52.2 in.
Rudder Area	5.752 sq. ft.
Rudder span (maximum)	49.5 in.
Rudder travel	
right (from streamline with fin)	16°
left (from streamline with fin)	16°
<u>Fuselage</u>	
Maximum width	40.0 in.
Maximum height	51.0 in.
Length (tip of nose to tip of tail)	256.50 in.
<u>Engine</u> (Continental)	
Type	C 85
Number of Cylinders	4

Power Settings	Condition	RPM
	Take off	2575
	MRP	2575
	Cruise	2400

<u>Propeller</u>

Manufacturer	Sensenich Bros.
Type	Wood, fixed pitch
Diameter	79 in.

INSTRUMENTATION

The instrumentation required to obtain the necessary data for this investigation consisted of an airspeed indicator, an altimeter, an accelerometer, and an elevator position indicator. The individual instruments may be described as follows:

1. Airspeed Indicator:

The airspeed was measured with a standard sensitive type airspeed indicator connected to a full swiveling pitot-static head. This instrument was calibrated by means of the speed course method and a calibration curve is contained in the appendix.

2. Altimeter:

The altitude was measured with a standard sensitive type altimeter.

3. Accelerometer:

Since the range of accelerations encountered in the tests was small and since it was desired to measure increments of acceleration as small as 0.25 g, the scale of a standard accelerometer was considered to be inadequate. A special accelerometer was constructed and calibrated for this investigation.

This accelerometer consisted of a glass tube approximately 20 inches long, a small coiled spring, a steel weight, and an attachment for connecting the spring inside one end of the tube. Two additional weights were constructed for use in calibration. These two weights were in the ratios of two and three to the weight to be used in the instrument.

With the spring mounted in one end of the glass tube, the instrument was calibrated by securing the tube in a vertical position, hanging each

of the three weights on the spring in turn and carefully marking on the tube the equilibrium position of the weight. In this manner the tube was calibrated for accelerations of one, two, and three "g's", three "g's" requiring an extension of the spring of about ten inches. Assuming the spring constant to be linear, the fractional "g" positions were easily located and inscribed on the tube.

A photograph of this accelerometer as mounted in the airplane may be seen in the Appendix.

4. Elevator Position Indicator:

The elevator position was measured by a 26 volt, 400 cycle a-c autosyn type transmitter. This transmitter was linked to the elevator control cable as close to the elevator horn as practicable. A calibration of the autosyn in terms of elevator angle is contained in the Appendix.

All data from the flight tests was visually read and manually recorded by the pilot and co-pilot.

WEIGHT AND BALANCE

All flight tests were made for three center of gravity locations, namely, 24.6%, 27.6%, and 30.6% mac. The normal center of gravity location for the airplane less pilots is 27.4% for a gross weight of 1100 pounds. The weight of the two pilots and parachutes was 335 pounds located at the center of gravity in all three cases. The normal operating gross weight for this airplane is 1450 pounds. The airplane was ballasted to the desired centers of gravity by means of lead weights attached either to the tail wheel arm or to the engine mounts as required. As all tests were flown with the fuel tanks approximately full, and since all tanks are located near the normal center of gravity, no corrections to center of gravity position were made to account for fuel consumption.

RESULTS AND DISCUSSION

I. GENERAL

The flight test data required for computing the two desired stability derivatives, the elevator power and the damping in pitch, is essentially the same in both cases. Determination of the elevator power, C_{m_δ}, requires curves of δ_e versus V_i at 1 g for several center of gravity locations. Determination of the damping in pitch, $C_{m_{d\theta}}$, requires these same curves for at least one center of gravity location (preferably several) and, in addition, the curves must be obtained not only at 1 g, but for several values of acceleration, i.e., several values of n.

The data was taken for three center of gravity locations as indicated under "Weight and Balance". The speed range over which the tests were run extended from 60 mph, V_i, to 120 mph, V_i. For the 1 g curves, data was taken in steady unaccelerated flight at increments in airspeed of 5 mph. For values of n greater than 1.0, the data was taken in accelerated flight in steady banked turns at increments of 10 mph in airspeed.

The airspeed instrument error was negligible; therefore V_i was equal to V_c. All flight tests were made at a pressure altitude of 5000 feet and since the speed range was low, the effects of compressibility were considered to be negligible and V_c was taken equal to V_e. The flight test data is plotted in Figures 1, 2, and 3 as curves of δ_e versus V_e.

In order to eliminate the effects of the elevator tab on required elevator deflection, and also to eliminate tab corrections in the calculations, the elevator tab was trimmed to zero deflection throughout the tests.

The greatest source of error (as will be shown later) in determining the desired derivatives, came in through the variation in the power effects. Since the Cessna 140 is equipped with a fixed pitch propeller it was impossible to maintain a constant power setting while varying the speed. At the higher end of the speed range (100 - 120 mph) it was necessary to reduce power rather drastically to avoid over-speeding the engine. The RPM was held constant throughout the tests. The cruising RPM of 2400 was chosen as the setting and was held at this value for all runs.

The succeeding sections treat the two derivatives under consideration in some detail in attempting to show how the value of the derivative is determined from reduction of the flight test data and how this value compares in magnitude with the theoretically computed derivative. Sources of error are analyzed and finally, an evaluation of the technique for obtaining the particular derivative is given.

II. THE ELEVATOR POWER, C_{m_δ}

1. Theory:

When an airplane is in equilibrium the sum of the moments acting on the airplane about any chosen reference line must be zero. If, for instance, the root leading edge of the wing is selected as the datum, or reference line, then the moment of the airplane's weight, W, about this datum is the product of the weight and the distance from the reference line to the center of gravity of the airplane (assuming the weight of the airplane to be concentrated at the center of gravity). This moment, then, is given by:

$$M_{ref.} = W \cdot x_{c.g.} \tag{1}$$

Now, if a new center of gravity position is obtained by shifting some portion of the airplane's weight (ballast weight = W_b) to a position aft of the original center of gravity by a distance, d, the moment of the airplane's weight about the reference line is given by:

$$M_{ref.} = W \cdot x_{c.g._2} \tag{2}$$

Therefore:

$$W \cdot x_{c.g_2} = (W - W_b)\, x_{c.g._1} + W_b\,(d + x_{c.g._1})$$

$$W \cdot x_{c.g._2} = W \cdot x_{c.g._1} - W_b \cdot x_{c.g._1} + W_b \cdot d + W_b \cdot x_{c.g._1}$$

or:

$$W\,(x_{c.g._2} - x_{c.g._1}) = W_b \cdot d \tag{3}$$

Obviously, then, the product of the airplane's weight and the change in center of gravity position must equal the applied moment, i.e.:

$$W\,(x_{c.g._2} - x_{c.g._1}) = M_{applied} \tag{4}$$

In flight, any moment acting on the airplane in the longitudinal plane must be balanced by a moment produced by the deflection of the elevator, That is, the sum of the applied moment and the moment due to elevator deflection must be equal to zero. Therefore, the magnitude of the pitching moment coefficient produced by the elevator per degree deflection is a measure of the elevator power. The moment produced by the elevator is given by:

$$\Delta M_e = \frac{dM_e}{d\delta_e} \cdot \Delta\delta_e \tag{5}$$

Therefore:

$$W\,(x_{c.g._2} - x_{c.g._1}) + \frac{dM_e}{d\delta_e} \cdot \Delta\delta_e = 0 \tag{6}$$

At equilibrium the lift must be equal to the weight ($L = W$). Making this substitution and putting the equation in coefficient form by dividing by qSc:

$$C_L \left(\frac{x_{c.g._2}}{c} - \frac{x_{c.g._1}}{c} \right) = - C_{m_\delta} \cdot \Delta \delta_e \tag{7}$$

Solving for C_{m_δ}:

$$C_{m_\delta} = - \frac{C_L (x_{c.g._2} - x_{c.g._1})}{\Delta \delta_e} \tag{8}$$

where $X_{c.g.}$ is now given in per cent of the mean aerodynamic chord.

2. <u>Determination of</u> C_{m_δ}:

Examination of the expression for C_{m_δ} developed in section 1 above, indicates that C_{m_δ} can be computed (as a function of airplane lift coefficient) from data obtained in simple flight test procedures. This technique may be outlined as follows:

1. Curves of δ_e versus C_{L_a} are obtained for at least two center of gravity locations of the airplane.
2. For a given C_{L_a} determine from these curves the change in elevator angle required by the given shift in center of gravity.
3. Compute C_{m_δ} using equation (8).

It should be noted that the curves of δ_e shown in Figure 4 are plotted versus the airplane's lift coefficient which is, in general, not equal to the lift coefficient of the wing alone. The simple relation derived above for determining the elevator power, C_{m_δ}, assumes that $C_{L_a} = C_{L_w}$. That this is not necessarily true is easily recognized from the fact that the lift of the tail is a definite contribution to the total lift of the airplane and varies with elevator deflection. For C_{L_a} to remain a constant as the center of

gravity position is shifted requires that for a change in tail lift due to elevator deflection a corresponding increase or decrease in C_{L_w} must occur. The change in wing angle of attack necessary to produce this lift change must in turn be compensated for by a slight change in elevator deflection. It has been shown, however, that the errors introduced by assuming $C_{L_a} = C_{L_w}$ for normal configurations are not large and probably are well within the accuracy of the flight tests. (Reference 3). For this reason it was considered justifiable to assume $C_{L_a} = C_{L_w}$ throughout this investigation and, therefore, C_{m_δ} was calculated using the simplified approach as outlined above.

3. Results:

A curve of C_{m_δ} versus C_L was computed in the above manner as shown in Figure 5. Since three equally spaced center of gravity locations were used in the flight tests, the values of δ_e used in computing C_{m_δ} were taken as the average change in elevator angle between the curves of δ_e versus C_L for the three center of gravity locations. Theoretically, for equal incremental changes in center of gravity position these curves will be equally spaced. That is, the change in elevator angle required to maintain a given C_L will be a constant for even increments of center of gravity shift. Also, theoretical curves of δ_e versus C_L for various center of gravity locations will intersect at a common elevator deflection at $C_L = 0$. It will be noted that the experimental curves of Figure 4 do not follow the theory exactly, particularly when C_L is less than 0.4. The curves obtained for the aft and mid center of gravity locations tend to follow the theory closely. However,

for lift coefficients below 0.4 the curve for the forward center of gravity exhibits a tendency to break down. For this reason the elevator power, C_{m_δ}, was computed only in the range of airplane lift coefficient from 0.40 to 1.20.

The discrepancy found in the forward center of gravity curve was also exhibited in previous flight tests of the Cessna 140. (Reference 2). The decrease in required elevator angle at the higher speeds may be due to one or more of several possibilities. It has been shown (reference 3) that as the center of gravity is moved further and further forward, the elevator increment per unit center of gravity change increases. However, the deviation of the curve is rather abrupt below $C_{L_a} = 0.40$ and it is believed that the discrepancy may be due in whole, or in part, to certain aero-elastic effects not presently accounted for. (Stabilizer deflection is among the possibilities.)

4. Theoretical Calculation of C_{m_δ}:

For an airplane flying in equilibrium (propeller off) the balance of moments (in coefficient form) is given by:

$$C_{m_{c.g.}} = C_{m_{a.c.}} + C_L \cdot \frac{x_a}{c} + C_{m_{fus.}} - \left(\frac{dC_L}{d\alpha}\right)_t \cdot \alpha_t \bar{V} \eta_t \qquad (9)$$

(Reference 1.) Differentiating this expression with respect to δ_e:

$$\frac{dC_m}{d\delta_e} = -\left(\frac{dC_L}{d\alpha}\right)_t \bar{V} \eta_t \frac{d\alpha_t}{d\delta_e}$$

$$\frac{d\alpha_t}{d\delta_e} = \tau_e = \text{elevator effectiveness} \qquad (10)$$

$$\therefore \quad C_{m_\delta} = -a_t \cdot \bar{V} \cdot \eta_t \cdot \tau_e \qquad (11)$$

This expression contains no propeller or power terms. This omission produces no error for the propeller wind-milling condition but will give results somewhat low for high power, low speed flight because of the increase in dynamic pressure at the tail due to the slipstream.

Calculation of the theoretical value of the elevator power according to this equation is as follows:

1. a_t: $A_t = \frac{b_t^2}{S_t} = \frac{(106)^2}{144 \times 24.35} = 3.2$

   This value for the tail aspect ratio is used to obtain the slope of the lift curve of the tail, a_t, from Ref. 1, Fig. 5-5.

   $$a_t = 0.048 \text{ per degree}$$

2. $\bar{V}$: $\bar{V} = \frac{S_t}{S_w} \cdot \frac{l_t}{c} = \frac{24.35 \times 12 \times 13.8}{159.29 \times 59.02} = 0.428$

3. η_t: Assume $\eta_t = 0.90$

4. τ_e: $\frac{S_e}{S_t} = \frac{9.66}{24.35} = 0.398$

   Using this value of the ratio of elevator area to horizontal tail area the theoretical value of τ_e is obtained from Ref. 1, Fig. 5-33.

   $$\tau_e = 0.59$$

Therefore: $C_{m_\delta} = -(0.048 \times 0.428 \times 0.90 \times 0.59)$

$$C_{m_\delta} = -0.0109$$

5. Comparison of Theoretical and Experimental Values for $C_{m\delta}$:

The theoretically determined value for the elevator power ($C_{m\delta}$ = — 0.0109) is plotted in Figure 5 for purposes of comparison with the experimental curve. The theory assumes $C_{m\delta}$ to be a constant based on the elimination of power effects. As pointed out above, this assumption leads to obvious inaccuracies in the low speed, high power condition due to the increase in the so-called tail efficiency, η_t. The tail efficiency is defined as the ratio of q at the tail to the free stream q. $C_{m\delta}$ varies directly as η_t and hence increases as C_L increases.

This variation of $C_{m\delta}$ with C_L is quite evident in the experimental curve, $C_{m\delta}$ varying from -0.0135 at $C_L = 0.4$ to -0.0167 at $C_L = 1.2$. Comparing these values with the theoretical value for this derivative, the experimental value is found to vary from 71.5% higher than theoretical for the low speed, high power condition ($C_L = 1.2$) to 23.8% higher than theoretical at cruising speed ($C_L = 0.4$).

As pointed out previously, however, with a fixed pitch propeller it was impossible to attain constant power settings over the speed range investigated. In order to avoid over-speeding the engine, power was rather sharply reduced at the higher speeds. Such reduction, obviously induces a greater variation in η_t than would normally be expected if the power could have been held constant. If this variation in the tail efficiency were accurately known and applied in making the theoretical calculation of $C_{m\delta}$ the discrepancy between theoretical and experimental values would tend to be reduced and probably would approach a constant as the lift coefficient increases. η_t varies somewhat linearly with C_L. Hence, applying this

correction to the tail efficiency would not affect the linearity of the theoretical curve.

The experimental determination of C_{m_δ} as performed under the assumption that $C_{L_a} = C_{L_w}$ also leads to values for the elevator power that are somewhat high. However, the use of this assumption yields an error that is usually within the accuracy of the flight tests.

6. Evaluation of Experimental Technique for Obtaining C_{m_δ}:

The stick-fixed trim curves are among the most accurate data that may be obtained from flight testing an airplane. These curves are usually reduced to plots of elevator angle versus lift coefficient, C_{L_a}, for several center of gravity locations. As the pilot technique required in these tests is simple, the accuracy of the data depends only on how accurately the elevator angle and the airspeed can be measured (assuming, of course, that the gross weight and the center of gravity location are accurately known). Excellent instrumentation is available for measuring these parameters.

The greatest errors involved in obtaining the elevator power with this technique come in through the power effects and the variation in η_t. For an airplane equipped with a controllable pitch propeller, the power can at least be held constant throughout the speed range of the investigation. Nevertheless, it is extremely difficult to accurately predict the variation in η_t and herein lies the greatest source of error. The problem is further complicated by the fact that for various airplane configurations the horizontal tail may be positioned such that it is completely in, partially in, or completely out of the slipstream. In the latter case, of course, the

problem is simplified since η_t is then a constant. For horizontal tails only partially submerged in the slipstream it is again difficult to assess just what portion of the tail is actually affected by the slipstream.

In spite of these errors, however, this flight test technique produces experimental values of C_{m_δ} that are in good agreement with theory and should prove to be a valuable tool in determining this parameter.

III. THE DAMPING IN PITCH, $C_{m_{d\theta}}$

1. Theory:

Consider an airplane flying along a straight path in equilibrium at some particular lift coefficient, i.e., unaccelerated flight. The balance of pitching moments is then given by:

$$C_{m_{c.g.}} = C_{m_o} + \left(\frac{dC_m}{dC_L}\right)_{C_L \cdot V^2 = K} \cdot \Delta C_L + C_{m_\delta} \cdot \Delta\delta_e \tag{12}$$

At each trim point the lift must be equal to the weight and hence $C_L \cdot V^2$ is a constant. If the airplane's speed is decreased its lift coefficient must increase and the change in pitching moments due to this increase must be balanced by the elevator. If the airplane is placed in a steady turn at some bank angle the lift must increase in order to provide a vertical component of lift equal to the weight. In a turn, however, the lift coefficient may be thus increased at no decrease in forward speed and the balance of pitching moments will be different from those obtained when the lift coefficient is increased in level flight. That is, the change in pitching moments, that must be balanced by a change in elevator deflection, will be different, and therefore the elevator deflection required to produce a given

increment in lift coefficient will not be the same as that deflection required to produce the same increment in lift coefficient in level flight. Obviously, this difference in elevator deflection is that increment required to overcome the damping in pitch of the airplane.

If the damping derivative is written in the form $C_{m_{d\theta}}$ where (d) is the operator $\frac{d}{d(t/\tau)}$ (τ = airplane's time characteristic = $\frac{m}{\rho S V}$), the balance of pitching moments for accelerated flight is given by:

$$C_{m_{c.g.}} = C_{m_0} + \left(\frac{dC_m}{dC_L}\right)_{V=K} \cdot \Delta C_L + C_{m_{d\theta}} \cdot d\theta + C_{m_\delta} \cdot \Delta\delta_e \qquad (13)$$

As shown in the section on elevator power, the elevator deflection must balance out any moments acting on the airplane if equilibrium is to be obtained. Having previously determined the elevator power, C_{m_δ} , from the 1-g trim curves, equation (13) offers a method of determining $C_{m_{d\theta}}$ from flight test data.

The airplane lift coefficient is defined as $C_{L_a} = \frac{2n(W/S)}{\rho V^2}$. At a given velocity, V, then, the lift coefficient on a 2-g curve is exactly twice that on a 1-g curve. If curves of $_e$ versus $V_{ind.}$ for several values of g are taken in steady turns and replotted as δ_e versus C_{L_a}, the increment in δ_e between two of these curves (between 1-g and 2-g for example) will be that increment in elevator deflection required to overcome the damping in pitch of the airplane. If no damping moments existed the curves of δ_e versus C_{L_a} for various g's would coincide since a given elevator deflection would produce a given lift coefficient at any acceleration. This assumes, of course, that the airplane's static stability does not vary with the different slipstream characteristics encountered at the various power settings used in obtaining the data.

If this assumption is made, these curves of δ_e versus C_{L_a} for various g's may be reduced to obtain the damping in pitch derivative, $C_{m_{d\theta}}$ in the following manner. The airplane's angular velocity in pitch may be shown to be:

$$\dot{\theta} \doteq \frac{g}{V}\left(n - \frac{1}{n}\right) \tag{14}$$

(Reference 1.)

Or, in terms of the operator (d):

$$d\theta = \frac{C_{L_a}}{2}\left(1 - \frac{1}{n^2}\right) \tag{15}$$

For any value of the airplane lift coefficient, then, $d\theta$ may be easily determined. The equilibrium condition is given by:

$$C_{m_{d\theta}} \cdot d\theta + C_{m_\delta} \cdot \Delta\delta_e = 0 \tag{16}$$

Therefore:

$$C_{m_{d\theta}} = -\frac{C_{m_\delta} \cdot \Delta\delta_e}{d\theta} \tag{17}$$

2. <u>Determination of</u> $C_{m_{d\theta}}$:

Equation (17) developed above indicates that $C_{m_{d\theta}}$ can be computed from flight test data obtained in accelerated flight. The technique may be outlined as follows:

1. Curves of δ_e versus C_{L_a} for several values of n, including n = 1.0, are obtained for one or more center of gravity locations of the airplane.
2. For a particular C_{L_a} and a particular n, compute $d\theta$, the airplane's rate of pitch in terms of the operator (d).
3. At this particular C_{L_a} determine the $\Delta\delta_e$ required to accelerate the airplane from n = 1.0 to the particular n under consideration.

4. Having previously determined C_{m_δ} as outlined in the section on elevator power, the damping in pitch, $C_{m_{d\theta}}$, is then easily determined from equation (17).

5. This process is repeated for a number of C_{L_a}'s and n's. Although, in general, $C_{m_{d\theta}}$ is not a function of lift coefficient, it is convenient to plot $C_{m_{d\theta}}$ versus C_{L_a} in order to establish an average value for the derivative. The scatter of the computed points is usually considerable. However, if a sufficient number of points are taken, the trend of the curve will be indicated well enough that the curve itself can be determined with fairly good accuracy. The final value for $C_{m_{d\theta}}$, then, is the average value of this curve.

3. Results:

The curves of δ_e versus V_e obtained in accelerated flight were replotted as curves of δ_e versus C_{L_a} as shown in Figures 6, 7, and 8. From these curves $C_{m_{d\theta}}$ was computed for every possible combination of C_{L_a} and n as outlined in the preceding section. A plot was then made of $C_{m_{d\theta}}$ versus C_{L_a} as shown in Figure 9. As expected, considerable scatter of the individual points was evident in this plot. However, the trend of the curve is clearly indicated. While the individual points computed are not shown in Figure 9, the range of the scatter is indicated by the cross-hatched band. As $C_{m_{d\theta}}$ is, in general, not a function of lift coefficient, the experimental curve of Figure 9 was considered to be constant over the range of lift coefficient investigated. The average magnitude was determined to be -0.195.

4. Theoretical Calculation of $C_{m_{d\theta}}$

The total damping in pitch of an airplane is the sum of the damping contributions of the various airplane components -- fuselage, wing, horizontal tail, and the propeller. In the normal configured airplane the damping due to the tail is by far the largest factor involved. It is the usual practice to evaluate the damping in pitch due to the tail and then increase the result by some arbitrary factor to take care of all the other contributions to the total damping. This arbitrary multiplying factor is normally taken equal to 1.10.

The damping due to the tail occurs as a direct result of the change in the effective angle of attack of the tail produced by the pitching velocity of the airplane, i.e., the angular velocity $\dot{\theta}$. This change in α_t is given by:

$$\Delta\alpha_t = \dot{\theta}\,\frac{l_t}{V} \tag{18}$$

The pitching moment produced by this angle of attack is:

$$C_m = -a_t\,\eta_t\,\bar{V}\,\frac{d\theta}{dt}\,\frac{l_t}{V} \tag{19}$$

Then:

$$\frac{dC_m}{d\left(\frac{d\theta}{dt}\right)} = -a_t\,\eta_t\,\bar{V}\,\frac{l_t}{V} \tag{20}$$

Dividing each side by the time characteristic, τ :

$$\frac{dC_m}{d\left(\frac{d\theta}{dt/\tau}\right)} = -a_t\,\bar{V}\,\eta_t\,\frac{l_t}{V\tau} \tag{21}$$

From which:

$$C_{m_{d\theta}} = -a_t\,\bar{V}\,\eta_t\,\frac{l_t}{c}\cdot\frac{1}{\mu} \tag{22}$$

To account for the rest of the airplane's damping in pitch multiply by 1.10 giving as the final expression:

$$C_{m_{d\theta}} = -1.10\, a_t \bar{V} \eta_t \frac{l_t}{c} \cdot \frac{1}{\mu} \tag{23}$$

Calculation of the theoretical value of the damping in pitch according to this equation is as follows:

1. a_t = 0.048 per degree = 2.75 per radian

2. $\bar{V}$ = 0.428

3. Assume η_t = 0.90

4. $\frac{l_t}{c} = \frac{13.8 \times 12}{59.02} = 2.8$

5. $\mu = \frac{m}{\rho S c}$

$$\mu = \frac{1450 \times 12}{(.00205) \times 159.29 \times 59.02 \times 32.2} = 28.0$$

$$\frac{1}{\mu} = \frac{1}{28.0} = 0.0357$$

$$C_{m_{d\theta}} = -1.1 \times 2.75 \times 0.428 \times 0.90 \times 2.8 \times 0.0357$$

$$C_{m_{d\theta}} = -0.1165$$

5. Comparison of Theoretical and Experimental Values for $C_{m_{d\theta}}$:

The theoretically determined value for the damping in pitch ($C_{m_{d\theta}}$ = - 0.1165) is plotted in Figure 9 to facilitate comparison with the experimental curve. The value of this parameter as deduced from the flight test

data is - 0.195 as shown in Figure 9. The experimental value is 66.8% higher than theoretical. This large discrepancy between experimental and theoretical results may be due to one or more of several factors. In the following paragraphs these possibilities are discussed in some detail.

In computing the experimental values of $C_{m_{d\theta}}$ at the various C_{L_a}'s, the magnitude of the elevator power, C_{m_δ}, used in equation 17 to determine $C_{m_{d\theta}}$ at a particular C_{L_a} was taken directly from the experimental curve of Figure 5. As pointed out in the discussion on elevator power, the experimental curve of C_{m_δ} versus C_{L_a} was also somewhat higher than theoretical. This discrepancy was attributed to the fact that the variation in η_t was unknown and to the assumption that $C_{L_a} = C_{L_w}$. It has been shown that if this assumption is not made, that is, the elevator power is computed on the basis of the lift coefficient of the wing alone when C_{L_a} is not equal to C_{L_w}, then the elevator power will be reduced to a lower value. (Reference 3) Since $C_{m_{d\theta}}$ varies directly with C_{m_δ} such a reduction obviously would also decrease the magnitude of $C_{m_{d\theta}}$.

Another important factor that has been assumed negligible in these calculations is the variation of the airplane's static stick-fixed longitudinal stability with the variation in power. It was assumed that the change in elevator angle required to accelerate the airplane to an n greater than 1.0 at a particular C_{L_a}, i.e., the $\Delta\delta_e$ between n curves, was entirely due to the damping in pitch of the airplane. It can be shown that:

$$\frac{d\delta_e}{dC_L} = -\frac{dC_m/dC_L}{C_{m_\delta}} \qquad (24)$$

That is, the elevator angle required to vary the equilibrium lift coefficient varies inversely with the elevator power, C_{m_δ} , and directly with the static stick-fixed longitudinal stability, dC_m/dC_L. The assumption, then, that the total $\Delta\delta_e$ between two different n's at a particular C_{L_a} includes the assumption that the static stability remains a constant throughout the test. This is probably a poor assumption in view of the fact that in some airplanes there is a wide variation in dC_m/dC_L with variation in power. (This variation is a function of the relative positions of the airplane's thrust line and its center of gravity location in the vertical direction). If this is the case, then, a wide variation in power as experienced in these flight tests may be expected to introduce considerable error in the value obtained for $C_{m_{d\theta}}$. If the variation of the static stability with power is known, or can be accurately predicted, such that the change in δ_e required by the static stability change can be deducted from the total $\Delta\delta_e$ required to accelerate at a given C_{L_a}, then $C_{m_{d\theta}}$ can be determined more accurately and its magnitude will be lower for the case where static stability increases as power is increased.

A third source of error which may account for part of the discrepancy between theoretical and experimental results lies in the theoretical calculation itself. It will be remembered that the damping in pitch of the airplane was considered to be due primarily to the horizontal tail damping. All other damping contributions due to the fuselage, wing, and propeller were lumped together as a multiplying factor of 1.10 to be applied to the horizontal tail damping. There is some evidence to indicate that this factor of 1.10 may be too low for airplanes having low relative densities.

Zimmerman developed a theoretical formula for $C_{m_{d\theta}}$ in 1935 in which this multiplier was taken to be 1.25. (Reference 4.) Since this time the relative density of most airplanes has increased considerably and the factor of 1.10 is generally considered adequate. For modern light airplanes with low wing loadings, however, the relative density factor, μ , remains low, which in itself tends to increase the damping parameter, $C_{m_{d\theta}}$, since $C_{m_{d\theta}}$ varies inversely with μ . Nevertheless, it is believed that increasing the multiplier from 1.10 to 1.25 or higher when working with light airplanes may lead to more accurate results in the theoretical calculation of $C_{m_{d\theta}}$. Increasing the multiplying factor to 1.25 yields a $C_{m_{d\theta}}$ for the Cessna 140 of $-$ 0.133. The experimental value obtained from the flight tests is then only 46.6% higher than theoretical as compared with 66.8% when the factor is 1.10. This new value, of course, is calculated under the same assumptions as before, i.e., the variation in C_{m_δ} and the change in static stability due to power are neglected.

6. Evaluation of Experimental Technique for Obtaining $C_{m_{d\theta}}$:

In the preceding sections a flight test technique for evaluating the airplane's damping in pitch has been outlined and discussed in some detail. The usual question now arises as to whether or not the method produces results of sufficient accuracy to be practical.

The pilot technique required to obtain trim curves in accelerated flight is relatively simple. The steady banked turn at constant speed is the easiest and most precise method of securing this data. The variables involved -- the airspeed, acceleration, and the elevator deflection -- can be measured with considerable accuracy. If the elevator power is accurately known, then,

it should be possible to reduce the flight test data to a reasonably exact value for $C_{m_{d\theta}}$.

The major problems to be solved in calculating $C_{m_{d\theta}}$ in this manner are the variation of C_{m_δ} with lift coefficient and the variation of static stability with power. The sources of error in determining C_{m_δ} have already been analyzed in previous sections. The static stability variation is particularly difficult with propeller driven airplanes. If the aircraft is jet or rocket powered there may still be important variations in the stability, but usually they are far less serious than for propeller driven airplanes. In high speed aircraft, however, there are additional problems arising from inaccuracies in the measurement of the extremely small elevator deflections required.

In spite of the difficulties encountered in separating the variables and minimizing the errors encountered, the value of this technique for obtaining the damping in pitch derivative should not be overlooked. In the past it has been necessary to rely on approximations of this parameter. At best, these theoretical approximations were in the nature of an "educated guess". Recently, more elaborate techniques for obtaining the longitudinal derivatives from the frequency response of the airplane have been developed. $C_{m_{d\theta}}$ can be determined in this manner but the method requires precision instrumentation and data reduction. Errors are cumulative throughout the procedure such that by the time the value of the damping derivative is reached the results are likely to be much less accurate than those obtained from the steady state flight tests. It is believed, therefore, that with good pilot technique in obtaining the required data, dependable instrumentation, and the elimination of as many of the

variables as possible, the steady state flight tests for determining the damping in pitch derivative as outlined will prove to be the most accurate method of obtaining this derivative for subsonic aircraft. Some refinements, such as a means of handling the static stability changes with power, are desirable and should be the subjects of future studies.

CONCLUSIONS

From the results of the flight test program conducted on the Cessna 140 airplane, it is concluded that:

1. The elevator power, $C_{m\delta}$, can be predicted with good accuracy from the unaccelerated trim curves for the airplane provided that:
   a). Power effects are carefully analyzed in order that errors due to variation in the tail efficiency, η_t, can be minimized.
   b). Consideration is given to the fact that the airplane lift coefficient may not be zero when the wing lift coefficient goes to zero. The assumption that $C_{L_a} = C_{L_w}$ is fairly good in most cases but the accuracy of the data reduction will be increased somewhat if the assumption is not made and the calculations are corrected accordingly.
2. The damping in pitch, $C_{m_{d\theta}}$, can be predicted with good accuracy from the accelerated trim curves for the airplane provided that:
   a). The elevator power, $C_{m\delta}$, can be determined accurately.
   b). The variation in static stick-fixed longitudinal stability with power is either small enough to be neglected or can be accurately predicted.
3. In making the theoretical calculation of $C_{m_{d\theta}}$ the multiplying factor of 1.10 used to correct the horizontal tail damping to include the damping of the whole airplane may be too low for airplanes having low relative densities.

RECOMMENDATIONS

On the basis of the above conclusions the following recommendations are made:

1. The unaccelerated trim curves of the Cessna 140 should be further investigated. In particular, the 1-g curve for the forward center of gravity location should be re-flown to check it below $C_{L_a} = 0.4$. The behavior of this curve should be carefully studied.
2. A further study of the elevator power of the Cessna 140 should be made in which C_{m_δ} is computed on the basis of the wing lift coefficient alone as outlined in reference 3.
3. A flight test program should be carried out on the Cessna 140 to determine the variation of static stick-fixed longitudinal stability with power in order that the computed value of $C_{m_{d\theta}}$ can be corrected for this effect.
4. A further study should be made in the theoretical determination of $C_{m_{d\theta}}$ to investigate the effect of low relative density of the airplane on the result. Damping in pitch due to the propeller, wing, and fuselage should be studied to determine more accurately what portion of the total damping must be attributed to the components.

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2. Graham, Dunstan, Longitudinal Stability and Control Flight Tests of the Cessna 140 Airplane, Aeronautical Engineering Laboratory, Princeton University, Report No. 111.
3. Perkins, Courtland D., Methods for Obtaining Aerodynamic Data Through Steady State Flight Testing, Part 1, The Longitudinal Derivatives, Aeronautical Engineering Laboratory, Princeton University, 1950.
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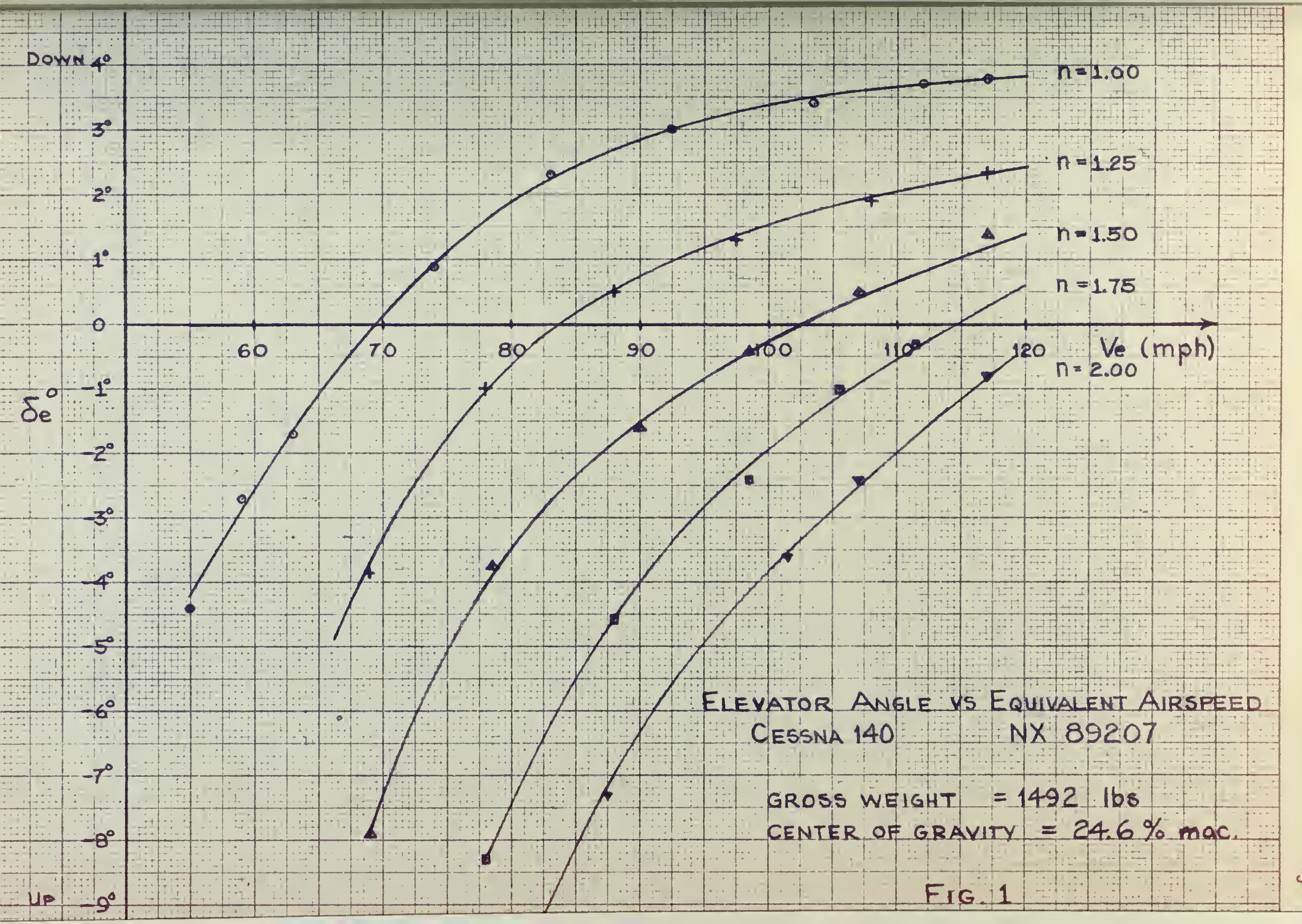
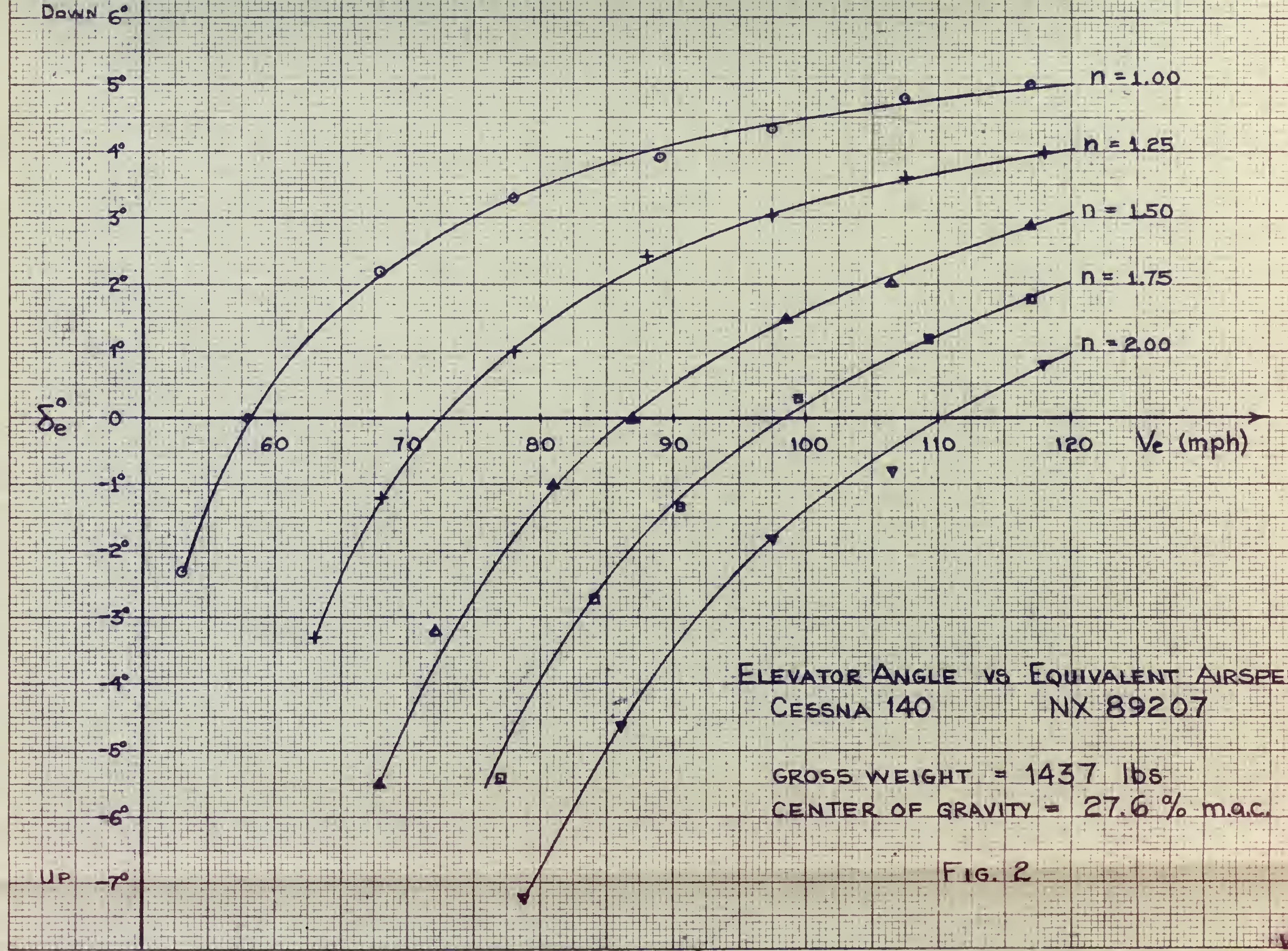

Elevator Angle vs Equivalent Airspeed
Cessna 140 NX 89207
Gross Weight = 1492 lbs
Center of Gravity = 24.6 % mac.
n = 1.00
n = 1.25
n = 1.50
n = 1.75
n = 2.00
Ve (mph)
60
70
80
90
100
110
120
δ_e°
Down 4°
3°
2°
1°
0
-1°
-2°
-3°
-4°
-5°
-6°
-7°
-8°
Up -9°

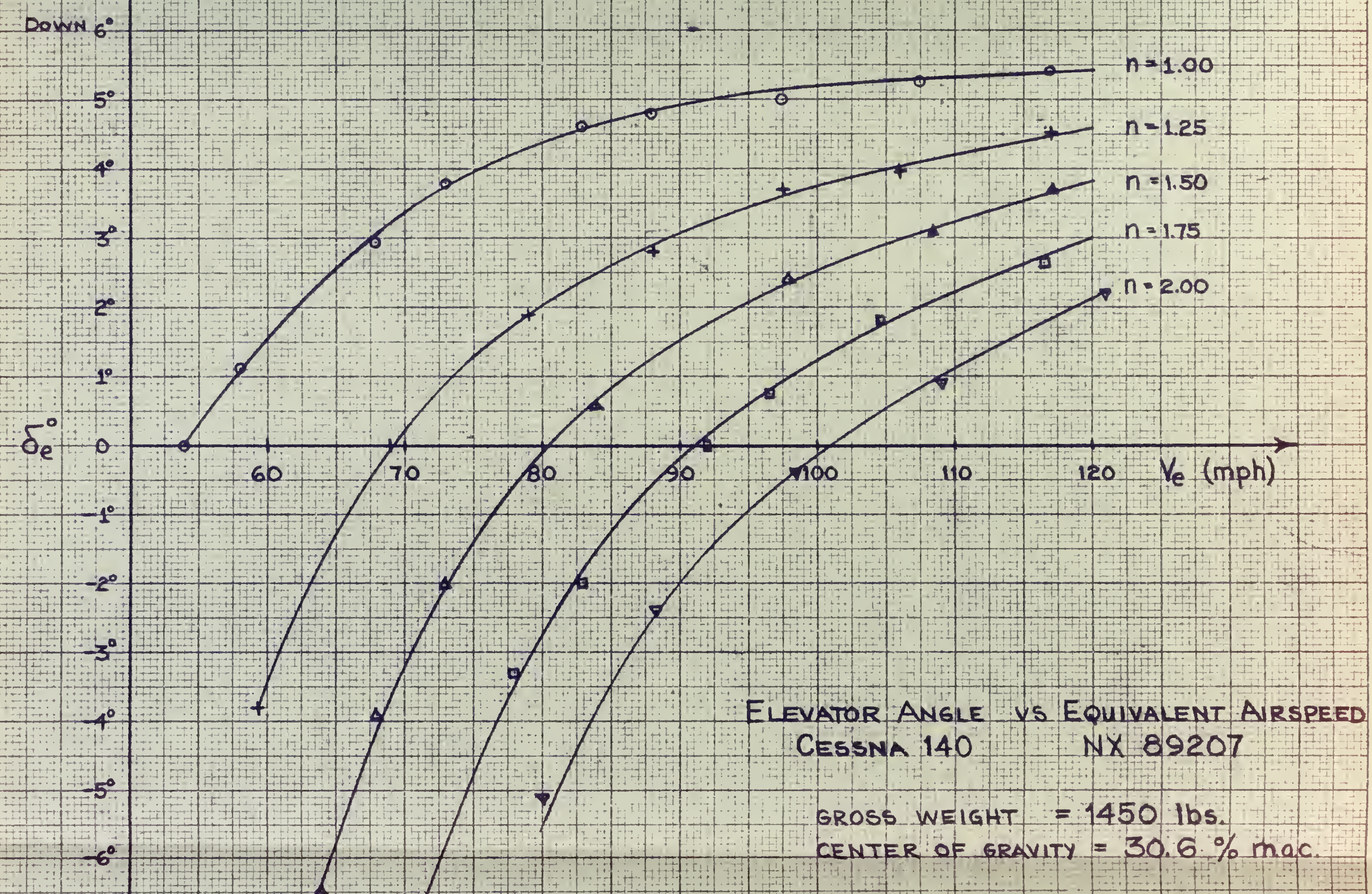
Fig. 1



Elevator Angle vs Equivalent Airspeed
Cessna 140 NX 89207

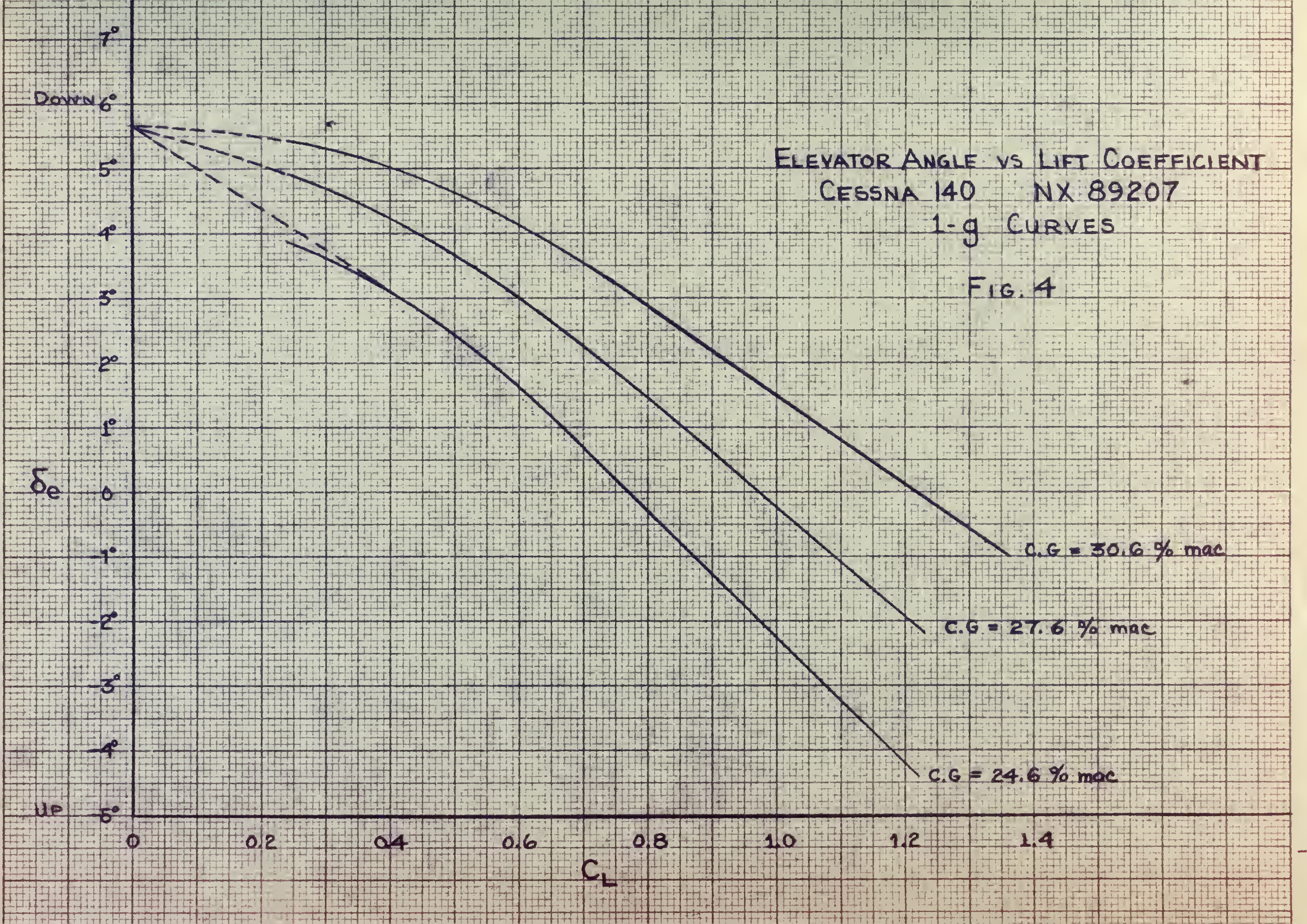
Gross Weight = 1437 lbs
Center of Gravity = 27.6 % m.a.c.

Fig. 2



ELEVATOR ANGLE VS EQUIVALENT AIRSPEED
CESSNA 140 NX 89207
GROSS WEIGHT = 1450 lbs.
CENTER OF GRAVITY = 30.6 % m.a.c.
n = 1.00
n = 1.25
n = 1.50
n = 1.75
n = 2.00
DOWN 6°
5°
4°
3°
2°
1°
0
-1°
-2°
-3°
-4°
-5°
-6°
UP -7°
$\delta_e°$
60
70
80
90
100
110
120
V_e (mph)

FIG. 3

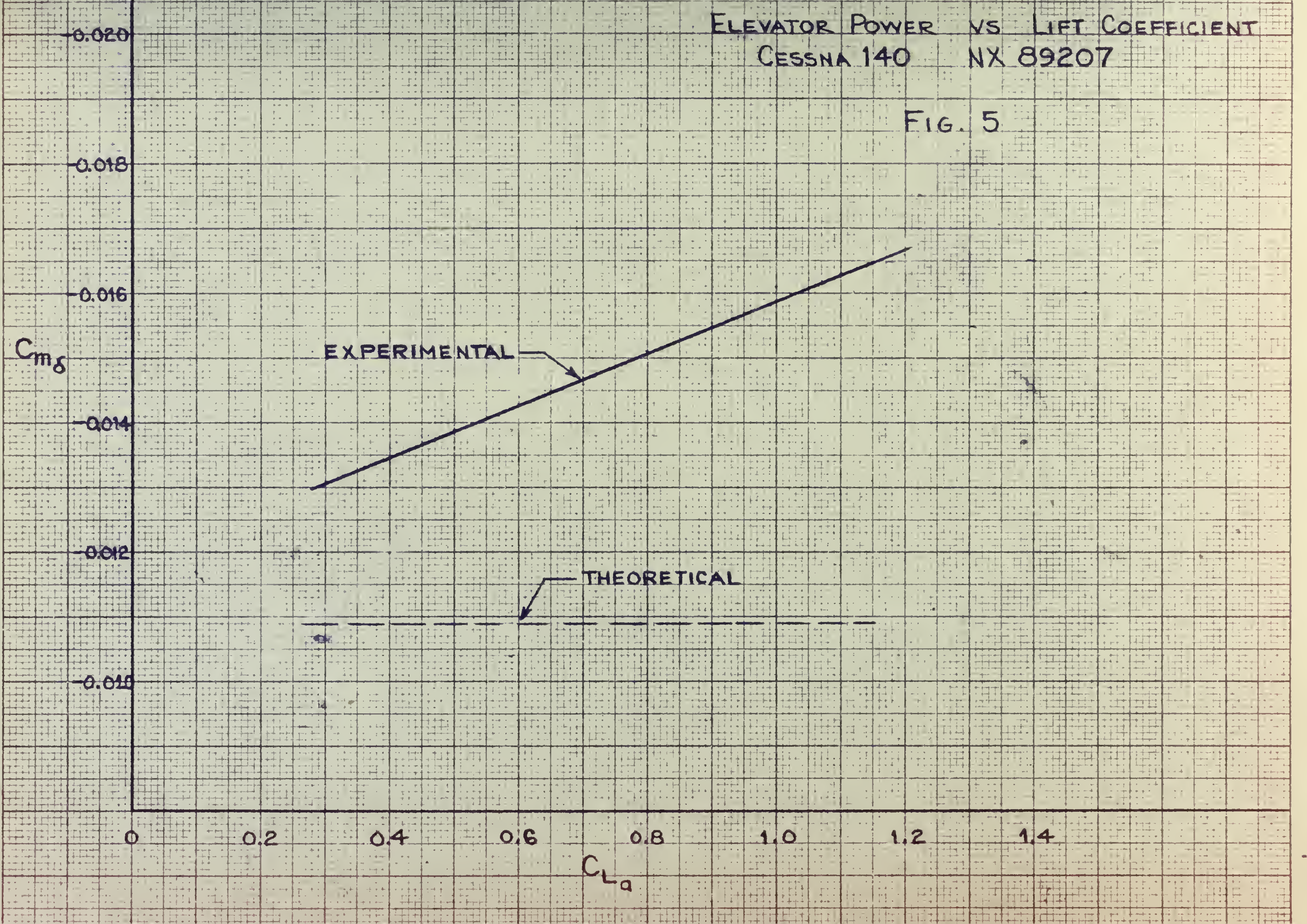


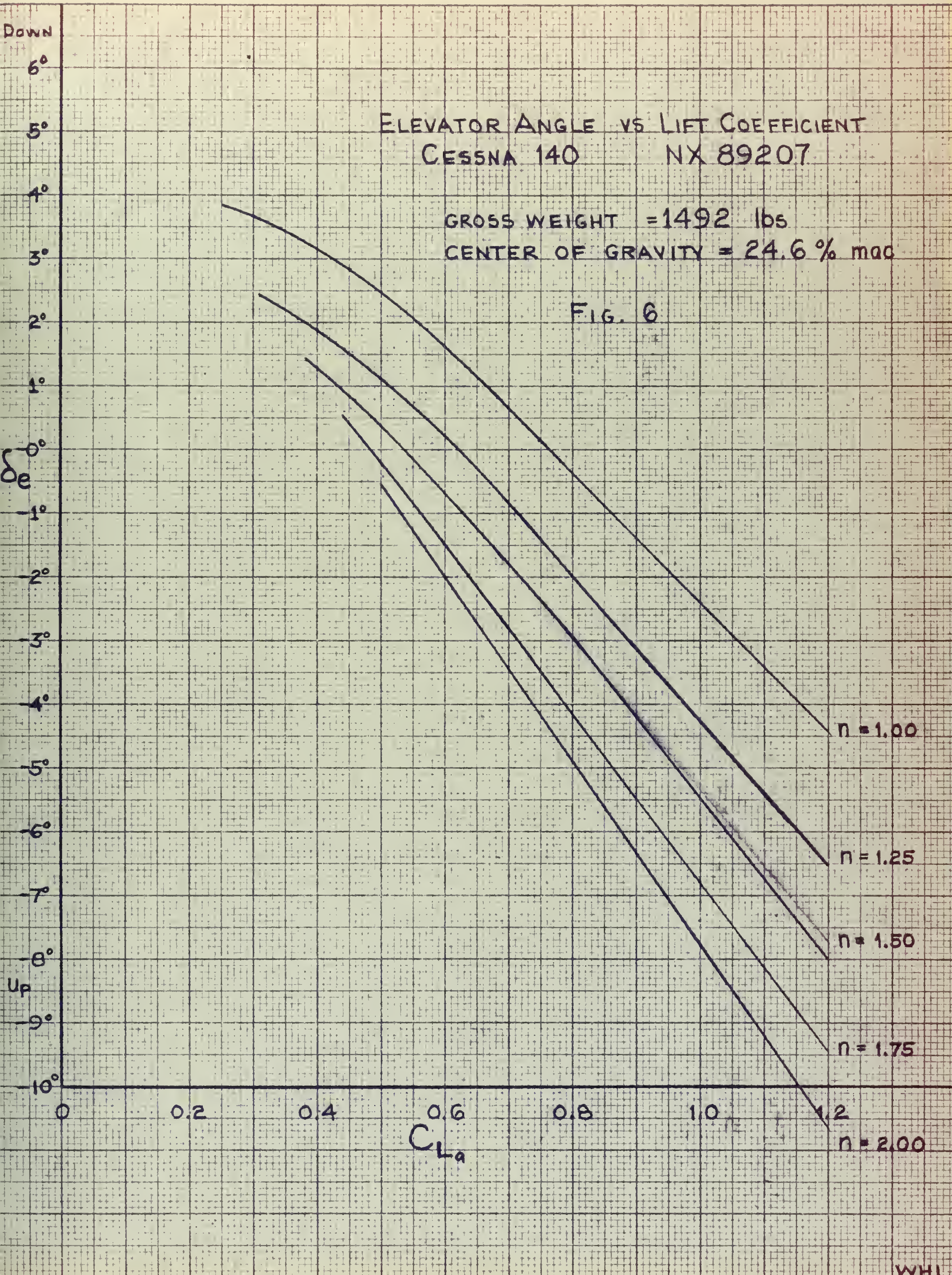
Elevator Angle vs Lift Coefficient
Cessna 140 NX 89207
1-g Curves

Fig. 4

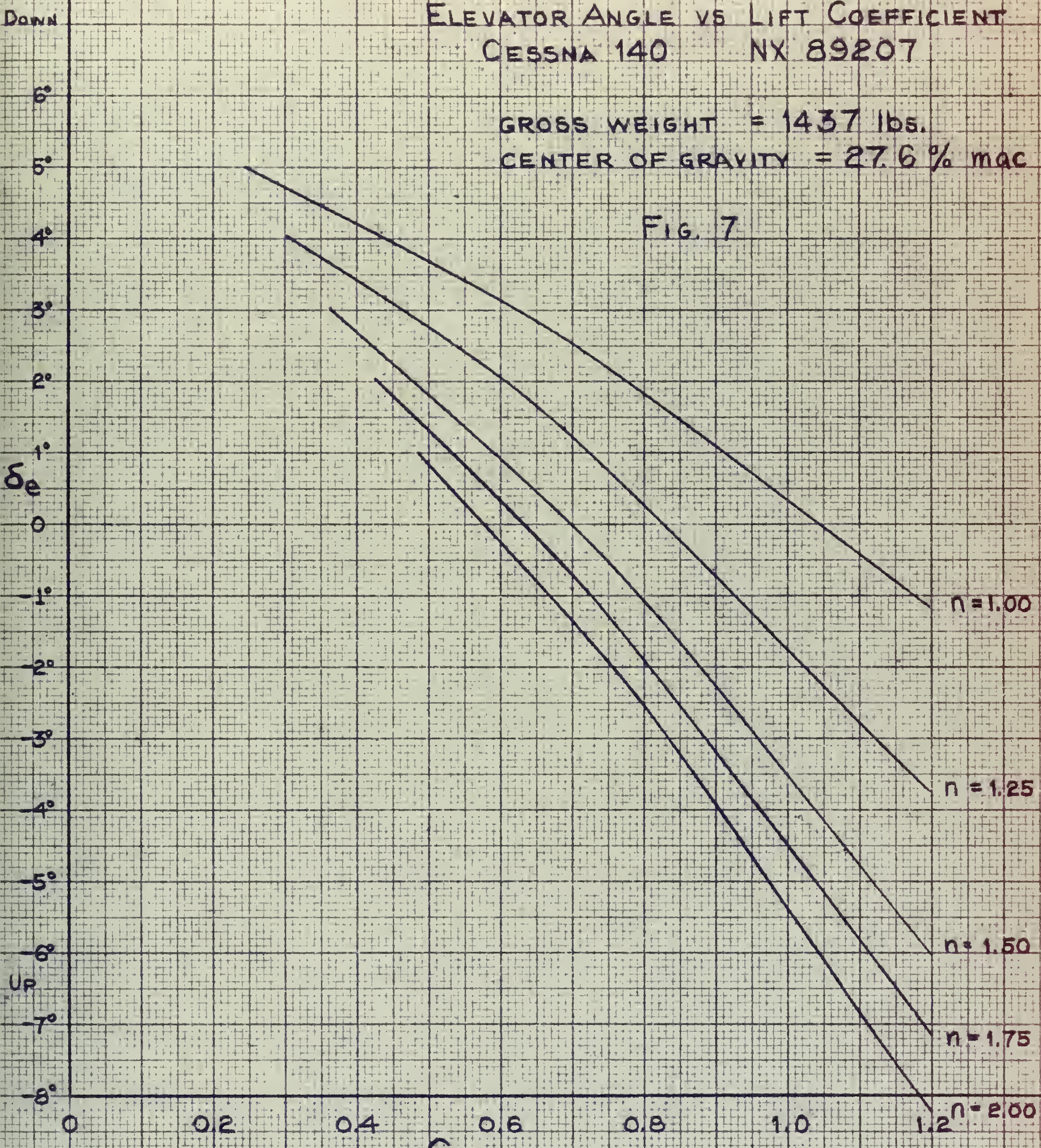
ELEVATOR POWER VS LIFT COEFFICIENT
CESSNA 140 NX 89207

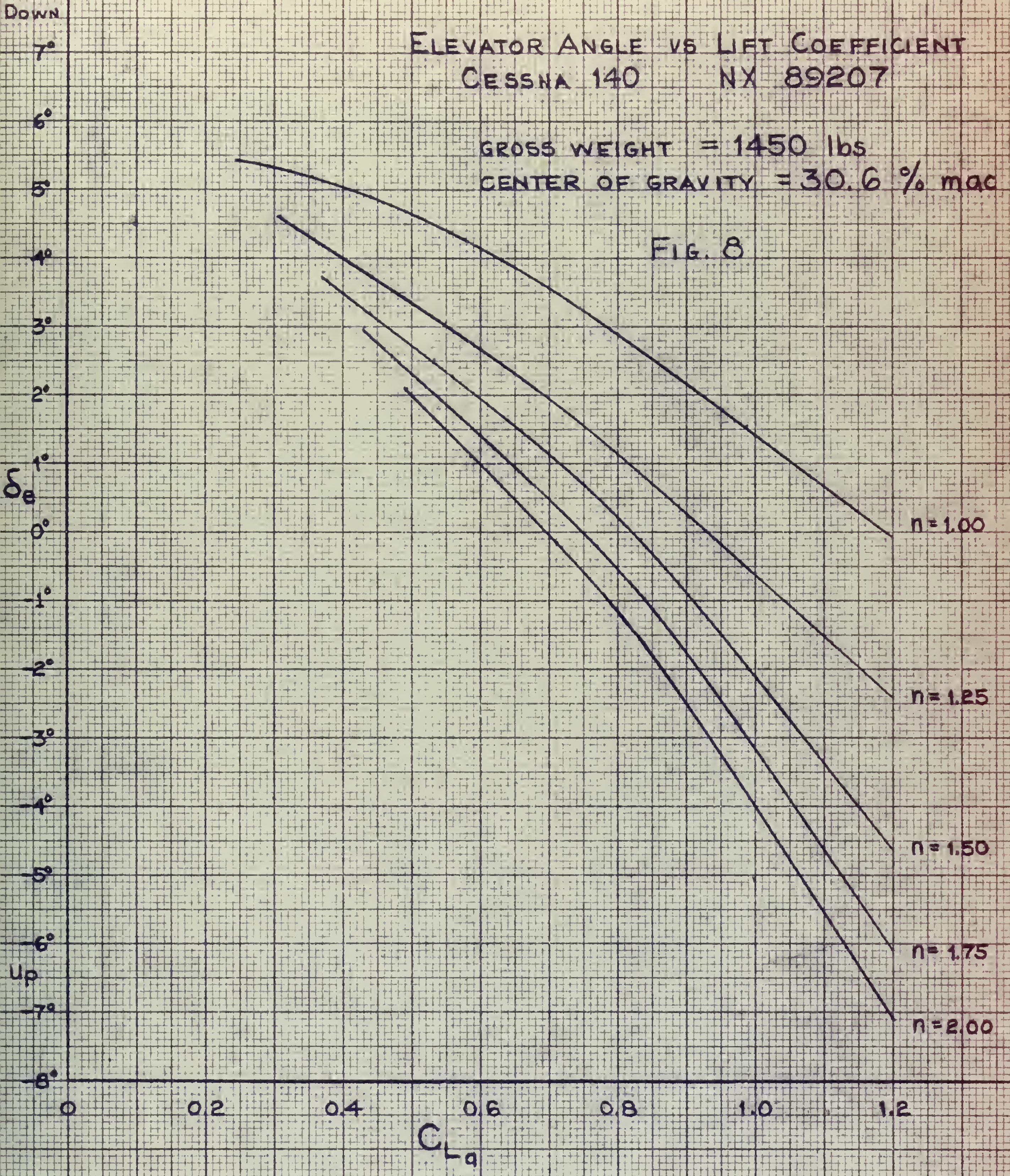
FIG. 5





ELEVATOR ANGLE VS LIFT COEFFICIENT
CESSNA 140 NX 89207
GROSS WEIGHT =1492 lbs
CENTER OF GRAVITY = 24.6 % mac
FIG. 6
DOWN
6°
5°
4°
3°
2°
1°
0°
δ_e
-1°
-2°
-3°
-4°
-5°
-6°
-7°
-8°
UP
-9°
-10°
0
0.2
0.4
0.6
0.8
1.0
1.2
C_{L_a}
n = 1.00
n = 1.25
n = 1.50
n = 1.75
n = 2.00
WHL

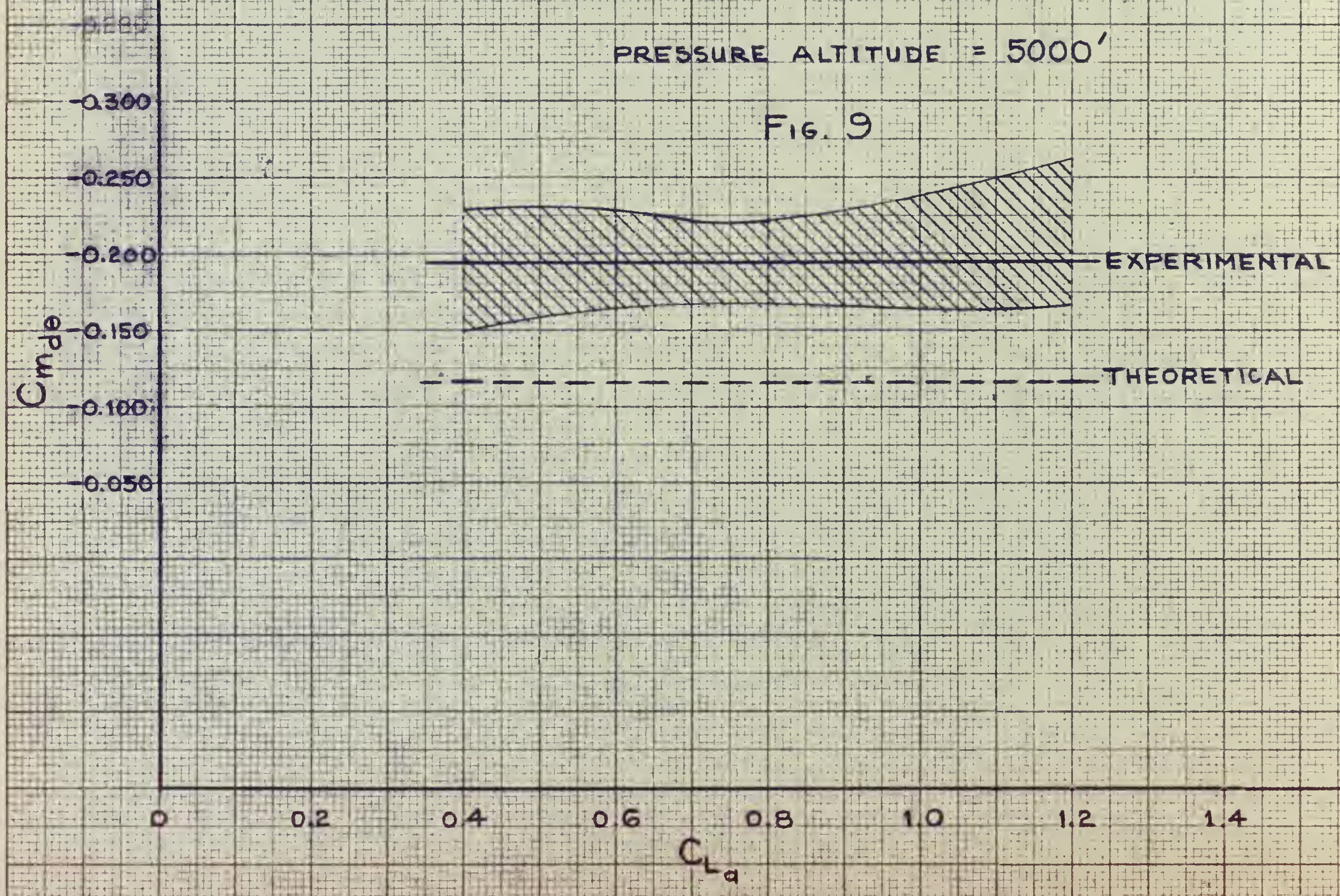

ELEVATOR ANGLE VS LIFT COEFFICIENT
CESSNA 140 NX 89207
GROSS WEIGHT = 1437 lbs.
CENTER OF GRAVITY = 27.6 % mac
FIG. 7
DOWN
UP
δ_e
6°
5°
4°
3°
2°
1°
0
-1°
-2°
-3°
-4°
-5°
-6°
-7°
-8°
0
0.2
0.4
0.6
0.8
1.0
1.2
C_{L_a}
n = 1.00
n = 1.25
n = 1.50
n = 1.75
n = 2.00


ELEVATOR ANGLE VS LIFT COEFFICIENT
CESSNA 140 NX 89207
GROSS WEIGHT = 1450 lbs
CENTER OF GRAVITY = 30.6 % mac
FIG. 8
DOWN
7°
6°
5°
4°
3°
2°
1°
δ_e
0°
-1°
-2°
-3°
-4°
-5°
-6°
UP
-7°
-8°
n = 1.00
n = 1.25
n = 1.50
n = 1.75
n = 2.00
0
0.2
0.4
0.6
0.8
1.0
1.2
C_{L_a}

DAMPING IN PITCH VS LIFT COEFFICIENT
CESSNA 140 NX 89207

PRESSURE ALTITUDE = 5000′

FIG. 9

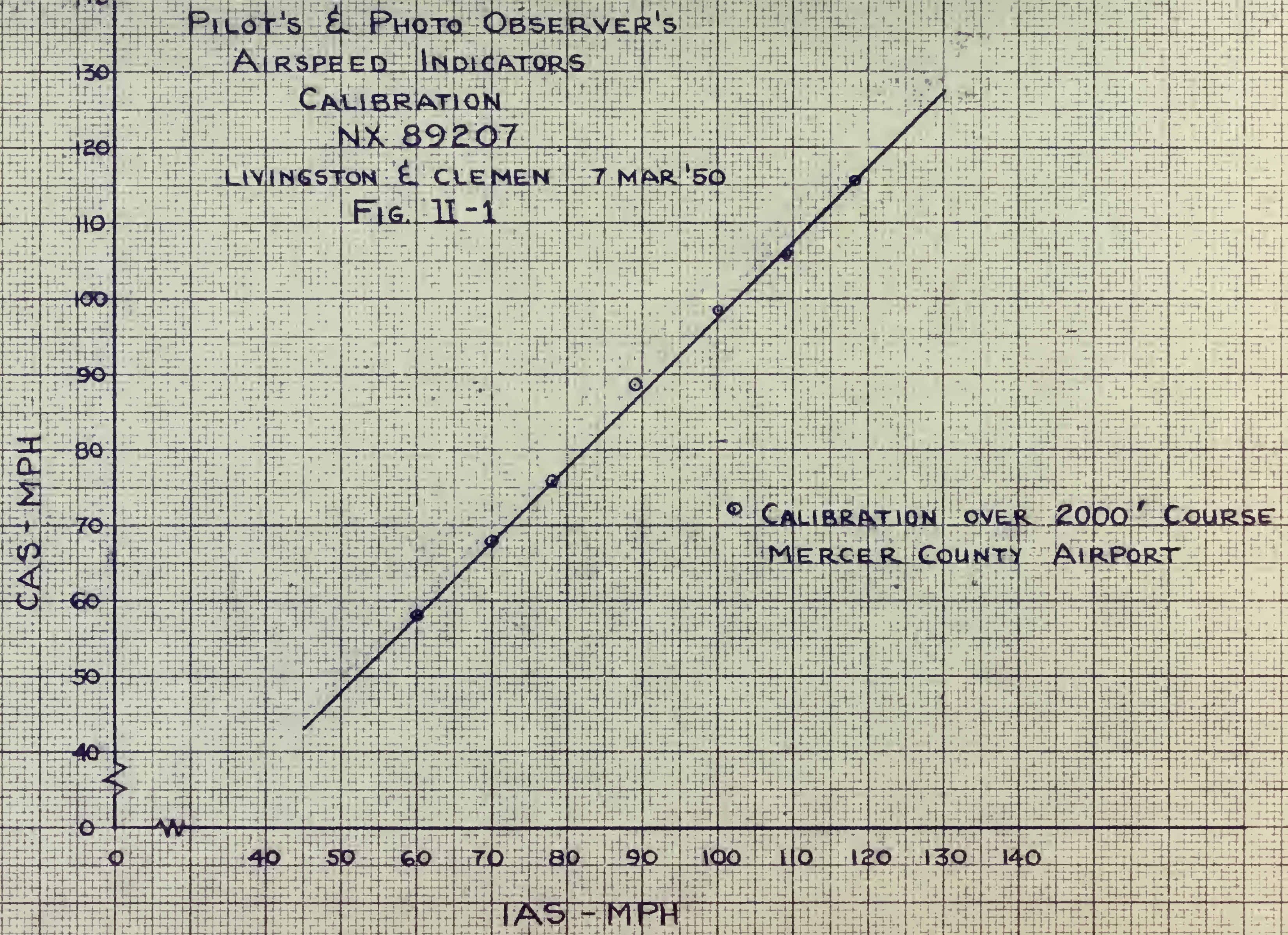


APPENDIX I

LIST OF SYMBOLS AND ABBREVIATIONS

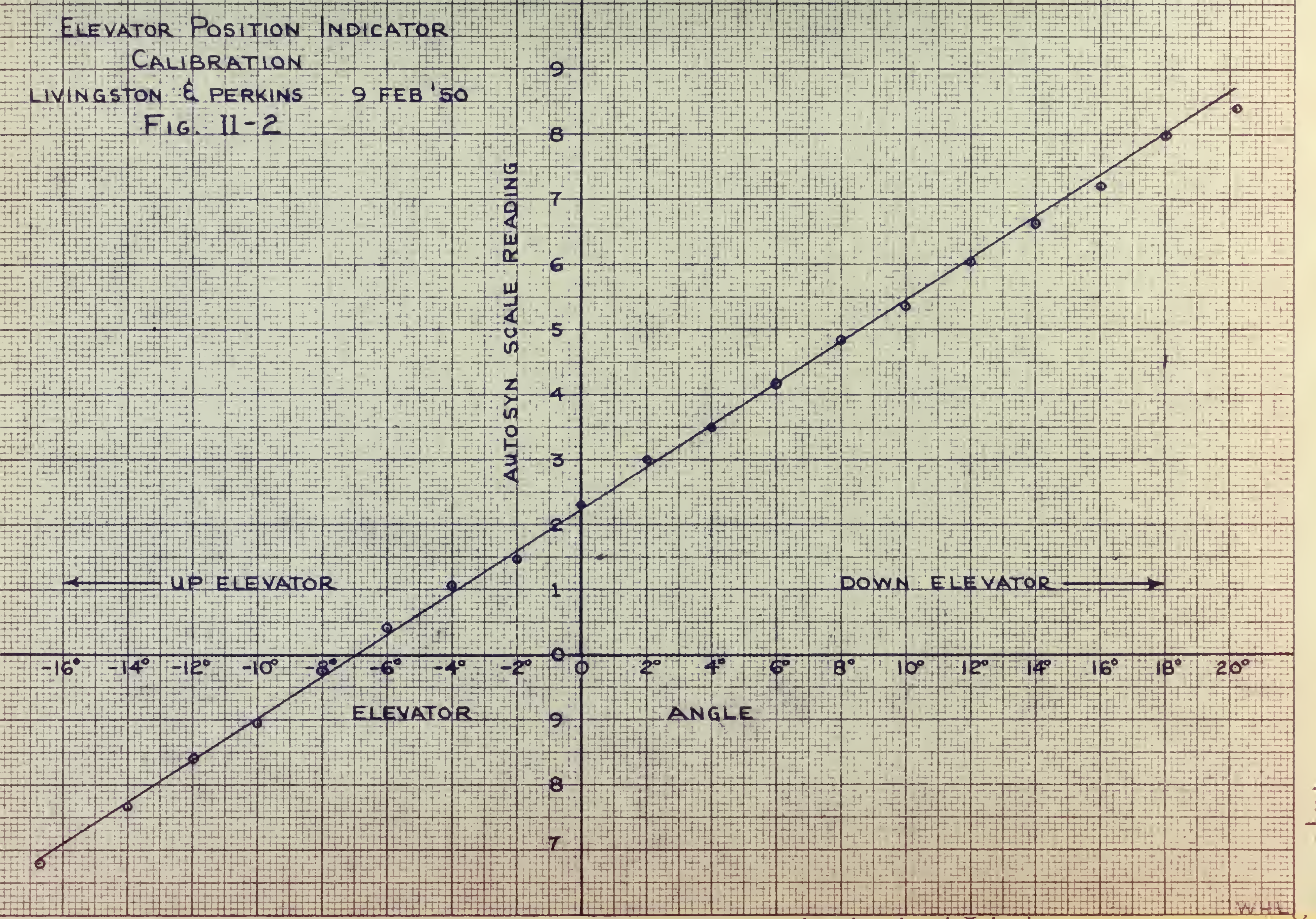
ρ	Air density, slugs/cu. ft.
V	True airspeed
V_i	Indicated airspeed
V_c	Calibrated airspeed
V_e	Equivalent airspeed
δ_e	Elevator deflection, degrees
q	Dynamic pressure, lbs/sq.ft. $= \frac{1}{2}\rho V^2$
g	Acceleration of gravity, 32.2 ft/sec/sec.
n	Normal acceleration in units of g.
c	Mean aerodynamic chord
S	Area
W	Weight
m	Mass $= W/g$
M	Moment
L	Lift
C_{L_a}	Airplane lift coefficient
C_{L_w}	Wing lift coefficient
a_t	Slope of horizontal tail lift curve
a_w	Slope of wing lift curve
α_w	Angle of attack of wing
α_t	Angle of attack of tail
$\bar{V}$	Tail volume coefficient $= \dfrac{S_t \cdot l_t}{S_w \cdot c}$
l_t	Tail length (airplane c.g. to aerodynamic center of tail)
η_t	Tail efficiency factor = slipstream q/ free stream q

τ_e	Elevator effectiveness $d\alpha_t/d\delta_e$
τ	Airplane's time characteristic $= \frac{m}{\rho S V}$
μ	Airplane's relative density $= \frac{m}{\rho S c}$
θ	Angle of pitch
$\dot{\theta}$	Angular rate of pitch
d	Operator $= \frac{d}{d(t/\tau)}$



Pilot's & Photo Observer's
Airspeed Indicators
Calibration
NX 89207
Livingston & Clemen 7 Mar '50
Fig. II-1
⊙ Calibration over 2000' Course
Mercer County Airport
CAS - MPH
IAS - MPH
0
40
50
60
70
80
90
100
110
120
130
140

ELEVATOR POSITION INDICATOR
CALIBRATION
LIVINGSTON & PERKINS 9 FEB '50
FIG. II-2



APPENDIX III

WEIGHT AND BALANCE COMPUTATION

Scale Calibration

Scale Serial No.	Load in Pounds											
	50	100	150	200	250	300	350	400	450	500	550	600
F 263218	151	201.5	251.5	301.5	351.5	401.5		502	552	602	652.5	702.5
F 263110	50.5	100.5	150.5	200.5	251	301	351	401		501.5	551.5	601.5
F 263111	44.5	100	150	200	250	300	350.5	400.5		501	551	601.5

Weight* and Balance

Condition: Oil and Fuel Tanks Full

Test Equipment Installed

1. **Weight**:

Scale	W'	Tare	Scale corr.	Weight
Left Wheel (F 263218)	632#	2.5#	-102.5#	527#
Right Wheel (F 263110	502#	1.5#	-1.5#	499#
Tail Wheel (F 263111)	127#	52.5#	0	74#
			Gross weight	= 1100#

* Airplane weighed in level flight attitude.

2. Balance:

Datum line (reference) . . . . Root leading edge

Mean aerodynamic chord . . . . 59.02"

Root leading edge to mean aerodynamic chord leading edge. . . . 0.47"

Datum line to main gear contact. . . . . 3.31"

Datum line to tail wheel contact . . . . 201.10"

a) Normal center of gravity location:

$$\bar{x}' = \frac{(W_L + W_R)\, D_m + W_t\, D_t}{\text{Gross weight}} \qquad \frac{(527 + 499)(3.31) + (74)(201.1)}{1100}$$

$$\bar{x}' = 16.64''$$

$$\bar{x} = \bar{x}' - \text{Dist. root L.E. to m.a.c. L.E.} = 16.64'' - 0.47 = 16.17''$$

$$\bar{x} = \frac{16.17}{59.02} = 27.4\% \text{ m.a.c.}$$

b) Weight of two pilots and equipment = 335 lbs.
Located at normal center of gravity location.

Weight of airplane and crew = 1100 + 335 = 1435 lbs.

c) Forward center of gravity location:

Added weight: 16 lead weights = 57.0 lbs. 26.5" forward of datum line.

$$\bar{x}_1 = \frac{(1435 \times 16.64) - (57 \times 26.5)}{1492} = 14.99''$$

$$\bar{x}_1 = \frac{14.99 - 0.47}{59.02} = 24.6\% \text{ m.a.c.}$$

d) Mid center of gravity location:

Added weight: 1 lead weight = 1.5 lbs., 191" aft of datum line.

$$\bar{x}_2 = \frac{(1435 \times 16.64) + (1.5 \times 191.0)}{1436.5} = 16.80''$$

$$\bar{x}_2 = \frac{16.80'' - 0.47''}{59.02} = 27.6\% \text{ m.a.c.}$$

e) Aft center of gravity location:

Added weight: 4 lead weights = 15.5 lbs., 191" aft of datum line.

$$\bar{x}_3 = \frac{(1435 \times 16.64) + (15.5 \times 191.0)}{1450} = 18.55''$$

$$\bar{x}_3 = \frac{18.55'' - 0.47}{59.02''} = 30.6\% \text{ m.a.c.}$$